Exercise 1

Reduce each of these quantities to a real number:

$$(a)\ \frac{1+2i}{3-4i}+\frac{2-i}{5i}; \qquad (b)\ \frac{5i}{(1-i)(2-i)(3-i)}; \qquad (c)\ (1-i)^4.$$

Ans. $(a)\ -2/5 \quad (b)\ -1/2; \quad (c)\ -4.$

Solution

Part (a)

$$\begin{aligned}\frac{1+2i}{3-4i}+\frac{2-i}{5i} &= \frac{1+2i}{3-4i}\cdot\frac{3+4i}{3+4i}+\frac{2-i}{5i}\cdot\frac{-5i}{-5i} \\ &= \frac{(1+2i)(3+4i)}{9-16i^2}+\frac{-10i+5i^2}{-25i^2} \\ &= \frac{3+4i+6i+8i^2}{25}+\frac{-10i-5}{25} \\ &= \frac{3+10i-8-10i-5}{25} \\ &= \frac{-10}{25} \\ &= -\frac{2}{5}\end{aligned}$$

Part (b)

$$\begin{aligned}\frac{5i}{(1-i)(2-i)(3-i)} &= \frac{5i}{(2-i-2i+i^2)(3-i)} \\ &= \frac{5i}{(2-3i-1)(3-i)} \\ &= \frac{5i}{(1-3i)(3-i)} \\ &= \frac{5i}{3-i-9i+3i^2} \\ &= \frac{5i}{3-10i-3} \\ &= \frac{5i}{-10i} \\ &= -\frac{1}{2}\end{aligned}$$

Part (c)

$$\begin{aligned}(1-i)^4 &= (1-i)(1-i)(1-i)(1-i) \\ &= (1-2i+i^2)(1-2i+i^2) \\ &= (-2i)(-2i) \\ &= 4i^2 = -4\end{aligned}$$